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AUTHOR Marshman, Bev
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ABSTRACT

This report compares the results of three successive second year calculus classes, in which part of each class was taught the concept of uniform convergence of a sequence of functions by an interactive COMIT module, while the remainder was given lectures in the traditional manner. An outline of the teaching techniques used in the module is followed by a comparison of the performances of the two groups on a pre-test, post-test, mid-term, and final exam. Finally, an attempt is made to evaluate the significance of any differences in the results, and to assess the assets and liabilities of computer aided teaching in general. (Author/JEG)

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A COMIT MODULE ON A TOPIC FROM ADVANCED CALCULUS

by

Bev Marshman

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A COMIT MODULE ON A TOPIC FROM ADVANCED CALCULUS

by

Bev Marshman

Department of Applied Mathematics

Abstract: This is a report on an experiment conducted with three successive second year calculus classes, in which part of each class was taught the concept of uniform convergence of a sequence of functions by an interactive COMIT module, while the remainder were given lectures in the traditional manner. An outline of the teaching techniques used in the module is followed by a comparison of the performances of the two groups on a pre-test, post-test, mid-term, and final exam. Finally, an attempt is made to evaluate the significance of any differences in the above results, and to assess the assets and liabilities of computer-aided teaching in general.

Introduction:

This project was very much a joint effort of myself and L. J. Makela, then a graduate student in the Department of Computer Science. Basically, I developed the course content, while he took charge of the programming, although there was much interaction at all stages. Since details of the software are included in his master's thesis [1], I would refer anyone interested in the computing aspects of the module's development to that source. In this report , I'll generally stick to a discussion of the teaching techniques we utilized.

Our principal aims were threefold. First, we wanted to see if the graphics capabilities and interactive nature of the COMIT facilities could be used effectively to convey the concept of uniform convergence of a sequence of functions to second year calculus students (irrespective of the effort and cost involved). Secondly, we wanted to compare the performance, both short-term and long-term, of the students who learned the topic through the COMIT module (approximately one-third of each group) with that of the remainder, to see if there were any detectable differences. Finally, we wished to assess, as far as possible, the advantages and disadvantages of this medium in general.

The Module:

The subject material of the module, uniform convergence of a sequence of functions, was chosen for a variety of reasons: it is most easily interpreted geometrically, and would thus utilize the graphics capabilities of COMIT; it is a concept which frequently gives students difficulty, and would, we hoped, truly test the system as a teaching device; finally, it is about the right length. The first stage was development of a set of behavioural objectives, which break the topic down into distinct steps, each building on the previous ones until the 'terminal objectives' are reached. These steps were then further dissected, examples selected, and a flow chart designed for each. Several months of programming followed, and the module was ready for use early in 1977. (For more details on all aspects of the structure of the module, see [1].)

The main teaching technique used was a combination of allowing the students to display for themselves the graphical behaviour of the functions involved while at the same time continuously soliciting answers to pertinent questions. In the following, one of the principal ideas contained in the module is presented (although not verbatim), along with explanations of what the student might have done and seen in that portion of the module; hopefully, this will clarify how the above-mentioned technique was utilized. (Anyone not interested in the actual content of the module can ignore the balance of this section, as it is not essential to the remainder of the report.)

A sequence of functions, denoted $f_n(x)$, is just a set of functions of a variable x, one for each value $n = 1,2,3,\ldots\ldots$. For example, consider the sequence $f_n(x) = x^n$, several terms of which are shown in Figure 1. Note that each term of the sequence is a single function of x.

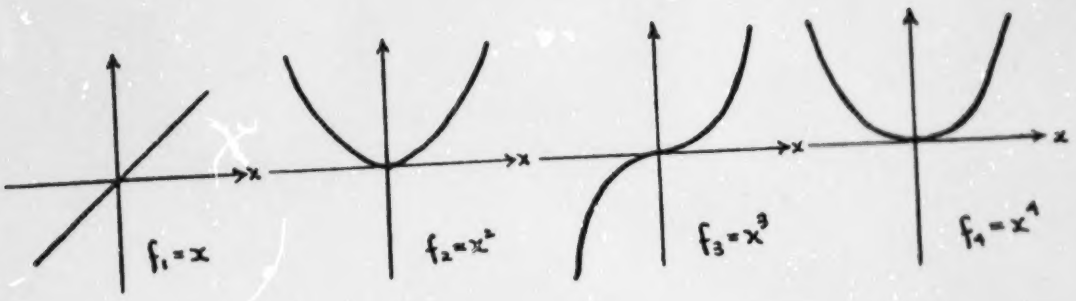

Figure 1

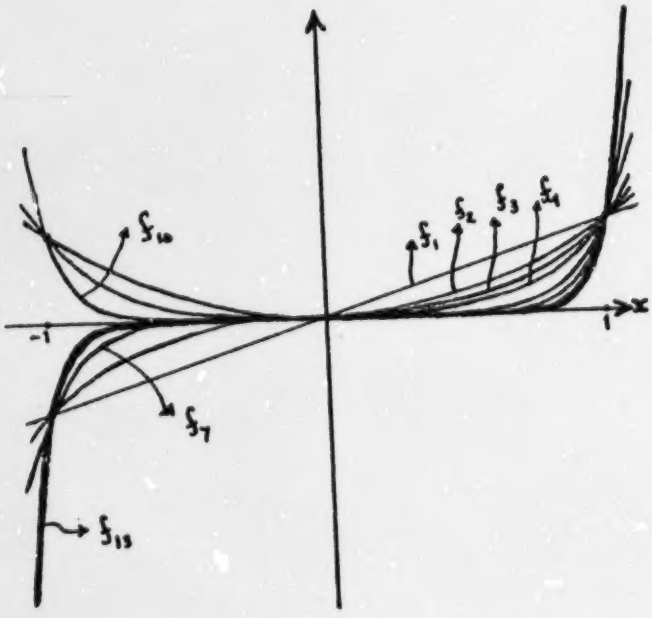

Figure 2

In the COMIT module, the student could choose to see any terms desired, but each was superimposed on the previous graphs, with the latest displayed in bright white and the former choices dimmed to a pale green. Thus a student who had asked, in turn, for terms n = 1,2,3,4,7,10 and 13 on the interval [-1.1,1.1] would, in the end, see Figure 2 displayed on the screen, with the graph of $f_{13}(x)$ displayed most brightly. (Note: To facilitate the fit, the vertical scale in this figure has been compressed.)

This example was then re-used to illustrate the notion of pointwise convergence: the sequence $f_n(x)$ converges pointwise at $x = x_0$ if the sequence of real numbers $f_1(x_o)$, $f_2(x_o)$, $f_3(x_o)$,..... approaches a certain value as n gets larger. For example, if $f_n(x) = x^n$, and $x_o = \frac{1}{2}$, we get the sequence $\frac{1}{2}, (\frac{1}{2})^2, (\frac{1}{2})^3, \ldots$, or $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{2^n}, \ldots$ which obviously approaches zero as n gets

large. (When this happens, we say the sequence has a limit and write, for example, $\lim_{n\to\infty} \frac{1}{2^n} = 0$.) On the other hand, if we choose $x_o = -1$, we get $(-1), (-1)^2, (-1)^3, \ldots$, or $-1,1,-1,1,-1, \ldots$ which does not approach any particular value as n gets larger. A moment's thought will convince you that the sequence x^n will only have a limit for values of x such that $-1 < x \leq 1$; i.e. either x is a fraction, so that x^n approaches 0 as n gets large, or $x = 1$, which gives the sequence $1,1,1,\ldots$. We say, then, that the sequence $f_n(x) = x^n$ has limit function

$$f(x) = \begin{cases} 0 & \text{if } |x| < 1 , \\ 1 & \text{if } x = 1 . \end{cases}$$

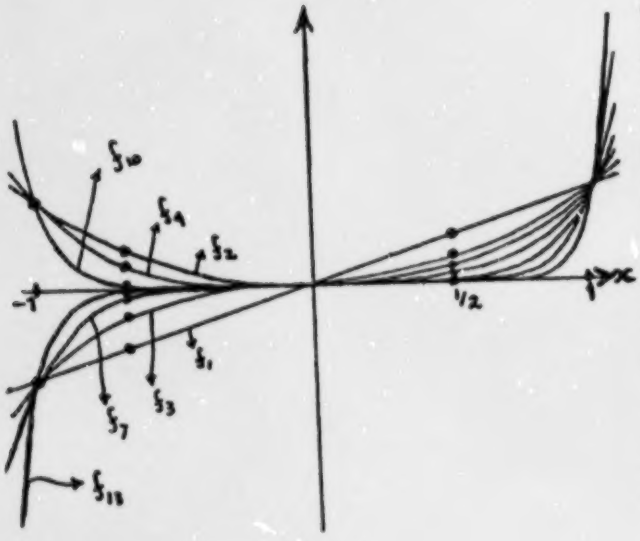

Figure 3

We again utilized graphics to illustrate this in the COMIT module; the convergent sequences for $x = \frac{1}{2}$, and $x = -\frac{2}{3}$, for example, are displayed as the vertical rows of dots in Figure 3. It is clear that they approach zero, while $x = -1$, say, oscillates between 1 and -1. (Here, 'approach zero' means the dots get arbitrarily close to the x-axis as $n \to \infty$.)

Once the concept of pointwise convergence had been presented, the student was allowed to 'play' with several other examples of sequences, using the graphics capabilities of the system to assist in finding for which values of x each sequence converged, and to what limit function $f(x)$. The student's grasp of the material was constantly monitored by a variety of types of questions (including some multiple choice), with immediate feedback giving the proper reasoning when errors

were made.

It should be pointed out that the above is a simplification of the actual material presented (mathematical rigour not being the object here); the aim was simply to demonstrate that the ideas could be handled graphically, and to give a hint of how this was done. (Complete flow charts of actual material are contained in [1].)

Structure of the Experiment:

From each of three different classes of students in Math 230B (Advanced Calculus), we selected ten students to use the module by first ranking them according to their Math 230A grades, then selecting every third student; the remainder of each group attended the usual two lectures on this topic, all being given basically the same two lectures by the author. All the students were given, unannounced, a pre-test just prior to the experiment (to measure their background in the relevant area), and a post-test, immediately following, to measure their grasp of the topic at that point. In addition, grades on the relevant questions on mid-term (except for group 2, where no mid-term was given) and final examination were recorded for each student, to serve as a measure of long-term retention of the material, although it should be noted that they are probably not very reliable indicators of any effect of the module or lectures encountered initially, since by that time many other factors will have come into play.

The students who used the module were given a set of notes which roughly paralleled the material in the module, except the interactive portions. These same notes were used as a basis for the lectures given to the remainder of each group, in hopes that all students would be exposed to more or less the same material.

Statistical Analysis:

I wish to thank G. W. Bennett of the Department of Statistics for the summary which comprises the balance of this section; further details on the

analysis may be obtained from him. Data used are the scores on the above-mentioned tests and exams, converted to percentages. Throughout the following, PRE and POST refer to pre-test and post-test grades, respectively, COMIT and NON-COMIT refer to the portions of each group using the module and attending lectures, respectively, and T1, T2, T3, and T4 refer to pre-test, post-test, mid-term, and final examination grades, respectively.

Table 1 illustrates the performances of COMIT and NON-COMIT on the pre-test and post-test for each of the three groups. The figures in brackets indicate the number expected in that cell under the hypothesis that there is no relationship between COMIT and POST > PRE. In all three groups, the figures are consistent with this hypothesis; that is, any differences are well within the allowable error.

An analysis of variance on -ℓn(grade) was done as a nested factorial with main factors COMIT, NON-COMIT, T1, T2, T3, T4, and students as a factor nested within the COMIT factor. This analysis shows a significant time effect for Groups 1 and 3, but with no significant differences between COMIT and NON-COMIT insofar as their patterns go. In Group 2, there were no significant effects, although the error terms were large enough to make differences difficult to detect.

Table 2 gives the means of grades on the various tests for each group.

In summary: (i) there is no evidence that COMIT versus NON-COMIT is related to the fraction of students with POST > PRE; (ii) there is some evidence that over time, the students learn, in that T3 is greater than T1 and T2, although again, there is no evidence that COMIT students behave any differently from NON-COMIT students in this respect.

It is important to note that the various marks measure performance under different conditions in that T1 and T2 are local to the

	GROUP 1			GROUP 2			GROUP 3		
TYPE OF STUDENT	NO. WITH POST > PRE	NO. WITH POST ≤ PRE	TOTAL (30)	NO. WITH POST > PRE	NO. WITH POST ≤ PRE	TOTAL (32)	NO. WITH POST > PRE	NO. WITH POST ≤ PRE	TOTAL (27)
COMIT	7 (6.7)	3 (3.3)	10	6 (5)	4 (5)	10	7 (6.7)	3 (3.3)	10
NON-COMIT	7 (10.3)	13 (9.7)	20	10 (11)	12 (11)	22	11 (11.3)	6 (5.7)	17
TOTALS	14	16	20	16	16	32	18	9	27

Table 1: Comparison of COMIT and NON-COMIT Students Results on Pre-test and Post-test. Three separate groups of students were involved.

	GROUP 1				GROUP 2			GROUP 3			
	T1	T2	T3	T4	T1	T2	T4	T1	T2	T3	T4
COMIT	.729	.821	.821	.780	.494	.594	.630	.538	.713	.787	.480
NON-COMIT	.607	.655	.869	.663	.632	.639	.564	.479	.686	.719	.414
COMMENT	This difference is mostly T3 improvement over T1 and T2.				No significant time pattern.			The bulk is due to the steady rise T1 → T2 → T3 and the sudden fall T3 → T4, although a significant T2-T1 component does not exist.			

Table 2: Comparison of Mean Grades for COMIT and NON-COMIT Students on Pre-test (T1), Post-test (T2), Mid-term Test (T3) and Final Exam (T4).

material, whereas T3 is only as a component of about half the course, while T4 is as a component of the whole course. Study preferences may influence T3 and T4.

Conclusions:

Our primary aim, that of testing the capabilities of COMIT as a teaching device for mathematical material, was fully achieved, and with great success. Aside from some complaints about the response time of the computer, over which we had no control, student reaction to the module was very favourable. The following is a selection of more or less typical quotes from the written comments handed in on completion of the module:

"I feel that seeing the graphs and being able to experiment with various n's and x's greatly helped me understand the concept of uniform convergence..... Another advantage of this teaching method is that it requires active participation of the student, where in a class, full attention is not always given."

"... I was forced to respond to questions all the time and thus take part in what was going on...."

"The graphical aspect certainly added to the comprehension..."

"Being able to go at your own pace and having things repeated when you didn't catch a concept was helpful. The use of examples with immediate feedback and explanations of how to do the questions was good also."

"The advantage was that I had to be on the alert all the time and thus I learned more about the topic. In class I normally feel sleepy sometimes ..... The idea of asking questions after each section was a good one. It tested the student's ability to retain the work."

Some of the students thought even more examples should have been included; some would have preferred two one-hour sessions rather than one two-hour, as their attention flagged towards the end. It should also be pointed out that the novelty of the situation probably contributed somewhat to their enthusiasm. I feel quite strongly, however, that the immediacy of the feedback is a really valuable asset of computer-aided instruction; we don't ever learn much by listening to someone talk about something; we learn by trying things out, making errors, correcting for them, and trying again. It is in this aspect that an interactive system excels.

Regarding our second aim, that of comparing the performances of COMIT versus NON-COMIT students, the statistical results speak for themselves: any differences are well within allowable error. The post-test grades, which are certainly the measures most closely related to the material presented in the module, were obtained on a ten-minute quiz, and it could be questioned whether such a quiz could test anything in depth. Nevertheless, it was designed to test whether their knowledge was at a level acceptable for that topic in that particular course, and as such represents as extensive a performance test as would ever be required. Keeping this in mind, we conclude that using COMIT did not produce perceptibly better performance (at least as we measure it) from the COMIT students over the NON-COMIT students. Whether that means they did or did not understand the concepts any better is quite another question!

Our third and final aim was to assess some of the advantages and disadvantages of the medium in general. The obvious ones have already been mentioned: the student sets the pace at which the material is covered; the constant stimulus provided by the interaction with the terminal keeps interest up, and accelerates assimilation of the concepts; the explanations given after each error provide a degree of 'personal' attention impossible in a lecture where there are more than half a dozen students. It is true that students using an interactive system like COMIT do not get the benefit of hearing the wider variety

of questions and answers, and discussion in general, that might arise in a lecture situation, but in most cases I think its other assets outweigh this, particularly since many courses nowadays are given in huge lecture sections where little or no discussion occurs.

On the negative side, we found the preparation time for this type of instruction to be astronomical (over 160 hours total for author and programmer); hopefully, this was at least partially attributable to inexperience. I certainly learned an awful lot about structuring a lecture, which was definitely worthwhile. It would also be nice to eliminate problems like slow response time related to backup computing facilities.. Finally, it should be pointed out that there has been no consideration of costs here. My general feeling is that, given the extensive effort necessary for preparation, my preference would be to use such a system for short modules on the more difficult topics of a course, with the students having access to these at their leisure.

All in all, I found the COMIT project most worthwhile and enjoyable, and at this point wish to extend my thanks to J. B. Moore, E. Elkeer, L. J. Makela, and the students themselves, for their co-operation and assistance.

Reference

[1.] Makela, L. J., CAI on COMIT: The Development and Implementation of a Computer-Assisted-Introduction Course Segment for use with Math 230B , Master's Thesis, Department of Computer Science, University of Waterloo, Nov. 1977.